Journal of the European Ceramic Society 20 (2000) 383–388

# On crack-growth resistance curve fitting for ceramics

Sergei M. Barinov*

*High Tech Ceramics Research Centre, Russian Academy of Sciences, Ozernaya 48, Moscow, 119361, Russia*

Received 30 March 1999; received in revised form 4 June 1999; accepted 3 July 1999

**Abstract**

$K_R$-curves of crack growth resistance were studied for a particulate ceramic-metal composite in the system lanthanium chromite–chromium in the temperature range 20 to 1100°C. It is shown that the $K_R$-curves can be described satisfactory by an exponential function. With the use of this function, the similarity of the crack-growth resistance curves for the specimens tested at different temperatures can be demonstrated. The notch-size effect can also be minimized if the $K_R$-curves are normalized with respect to crack length, the normalizing factor being the parameter $l$ of the exponential function. A possible background in the framework of the model of the crack-face bridging by frictional ligaments in a wake zone is proposed. 

*Keywords:* Composities; Crack growth resistance; Fracture; $LaCrO_3$/Cr

## 1. Introduction

Crack length dependence of crack growth resistance $K_R$, or crack-growth resistance curve behavior, is one of the most important subjects in the fracture mechanics evaluations of brittle ceramic-base materials. Many studies have been performed with $K_R$-curves in ceramics, the recent results being outlined in Ref. 1. Attempts were undertaken to relate the parameters of the $K_R$-curves to the micromechanisms of crack–microstructure interaction.[2]

To describe the $K_R$-curve behavior, a power function is commonly applied

$$\Delta K_R = A(\Delta a)^n \tag{1}$$

where $A$ is a constant; $\Delta a$ is the crack increment, and $0 < n < 1$.[3–5] This power function is a good approximation of $K_R$-curve, especially at the early stage of stable crack propagation. However, significant deviations from the power law can be expected at the final stage of the stable crack propagation process. Besides, the parameters $A$ and $n$ of Eq. (1) vary with initial notch depth, specimen size, etc., and therefore have not clear enough meaning to be related with the microstructural features of fracture. The problem of the non-invariance of parameters of Eq.(1) with respect to the testing conditions is still open. The aim of this study was to investigate the $K_R$-curve behavior in a particulate ceramic-composite material and to propose an alternative approach to describe this behavior, which could allow us to discuss the experimental data in relation to the micromechanisms of fracture.

## 2. Experimental

Experiments were performed with the specimens of a ceramic–metal composite in the system lanthanium chromite–chrome. This material is intended for application in severe thermal environments, e.g. in power energy installations, and have therefore to be thermal shock fracture resistant.[6] The specimens were prepared using powder mixtures of chromium (40 wt%) with a particle size of 1–20 μm and lanthanium chromite (60 wt%) with an average particle size of 20 μm. An explosive compacting method was employed followed by heat treatment; the details of the processing route were described elsewhere.[7]

The specimens of a standard SENB fracture mechanics configuration were of $B \times W = 4 \times 6\,\mathrm{mm}^2$ cross-section and 50 mm length. A thin side notch was machined in the specimens with a diamond saw-cut wheel. The depth of the notch was half of the specimen width, the measured curvature radius at the notch tip was about 50 μm.

* Tel.: +7-095-430-7770; fax: +7-095-437-9893.

0955-2219/00/$ - see front matter 
PII: S0955-2219(99)00191-0

The specimens were loaded in a stiff three-point bending ajustment at a span of 24 mm in a furnace filled with argon at a temperature 20, 700 and 1100°C. The cross-head speed was $8\times10^{-6}$ ms$^{-1}$.

The $K_R$-curves were constructed using the relation between the non-dimensional load, $P^*$, and non-dimensional displacement, $u^*$, under the load in the form

$$P^* = 0.00114u^*(1-2a^*)^2/[(1-1.85a^*)2+0.0013u^*]^{1/2} \tag{2}$$

where $P^* = P/\sigma_y W^2$, $u^* = Eu/\sigma_y W$, $a^* = \Delta a/W$, $P$ is the load, $\sigma_y$ is the yield stress, $E$ is the modulus of elasticity, $u$ is displacement and $\Delta a$ is the crack increment.[8] Eq. (2) allows us to calculate the crack increment through the changes in compliance, $u/P$, and no direct crack length measurements are needed in this method. The details of the use of Eq. (2) are described in Ref. 9.

The current $K_R$ values were estimated as

$$K_R = 1.5Y(\alpha)a^{1/2}PL/BW^2 \tag{3}$$

with the $K$-calibration polynomal $Y(\alpha)$ given by

$$Y(\alpha) = \left[1.99-\alpha(1-\alpha)\left(2.15-3.93\alpha+2.7\alpha^2\right)\right]/(1+2\alpha)(1-\alpha)^{3/2} \tag{4}$$

where $\alpha = a/W$ is reduced crack length.[10]

## 3. Results and discussion

Fig. 1 shows the $K_R$-curves for the composite at different temperatures. A least-squares fit of the data results in a power function of Eq. (1) which is a good approximation of the curves. The calculated parameters $A$ and $n$ of this equation are varying with testing temperature and equal to:

| | | | |
|---|---|---|---|
| Testing temperature (°C): | 20 | 700 | 1100 |
| $A$: | 1.08 | 0.89 | 0.85 |
| $n$: | 0.82 | 0.80 | 0.74 |

The exponent $n$ is approximately invariable with respect to testing temperature indicating the same fracture micromechanisms at different temperatures. However, the values of A and n cannot be easily interpreted quantitatively.

As an alternative to the power law of Eq. (1), let us consider an approximation of the $K_R$-curves by an exponential function as follows

$$\Delta K_R = (K_m - K_0)[1-\exp(-\Delta a/l)] \tag{5}$$

where $K_m$ corresponds to the plateau on the $K_R$-curve owing to the asymptotic form of Eq. (5); $K_0$ is the stress

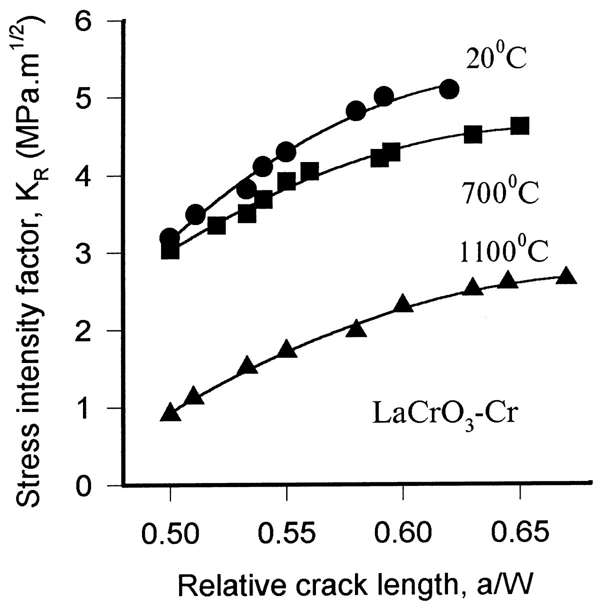


Fig. 1. $K_R$-curves for $LaCrO_3$–Cr composite tested at 20, 700 and 1100°C.

intensity factor at the start of the crack from the notch tip; $l$ is a normalizing factor.

It can be seen that the $K_R$-curves presented in Fig. 1 do not reach exactly a plateau. However, it can be assumed that the maximal stress intensity factor along each experimental $K_R$ curve is sufficiently close to a plateau value, just because the following crack propagation occurs in an unstable manner at a constant stress intensity factor value. It seems therefore to be reasonable to use these maximal values as scale parameters, at least in first approximation. To calculate $K_m$ more precisely, the common extrapolation methods could be employed.

Shown in Fig. 2 are the experimental $K_R$-curves replotted in coordinates $\ln[1-\Delta K_R/(K_m-K_0)] - \Delta a$. The slope of the plots gives the values of the normalizing factor:

| | | | |
|---|---|---|---|
| Testing temperature (°C): | 20 | 700 | 1100 |
| Normalizing factor $l$ (μm): | 160 | 240 | 250 |
| Scale parameter $K_m - K_0$ (MPa. m$^{1/2}$): | 1.62 | 1.12 | 1.03 |

The value of the normalizing factor is supposed to be a measure of the length of a steady-state crack-bridging zone.[11]

Using the values of the normalizing factor, the $K_R$-curves can be replotted as a function of the reduced crack increment, $\Delta a/l$ (Fig. 3). It becomes obvious, that a similarity of the crack-growth resistance curves takes place, because all the normalized curves are fitted well with an unique function. Some divergence occurs only at high crack increment values, probably due to the difference in $K_m - K_0$ for the specimens tested at different temperatures. Therefore, it can be supposed that the same mechanism of the crack-microstructure interaction is operated during the stable crack propagation in the composite material in the temperature range from 20 to 1100°C. It has been assumed that the stable crack growth in the composite is due to the existence of grains bridging the surface and restricting the crack-opening displacement.[11] The bridging stresses can principally result from the elastic, elastic–plastic and/or frictional ligaments in the crack-tip wake zone. It was approved that the major contribution to the bridging stress in the composite under investigation gives the debonded and frictionally sliding ligaments.[11] An increase in testing temperature from 20 to 700 and, further, to 1100°C results in a decrease in the tangential forces to debond and slip the bridging ligaments relatively, and to decrease therefore the mean value of the bridging stress.

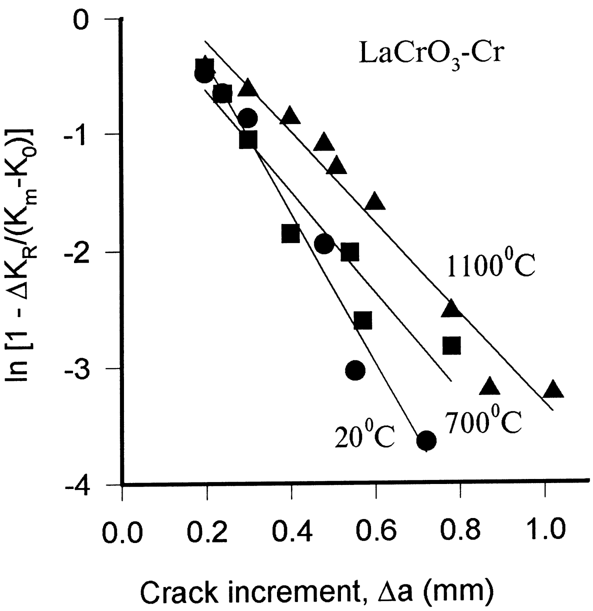


Fig. 2. Representation of $K_R$-curves for $LaCrO_3$–Cr composite by exponential function.

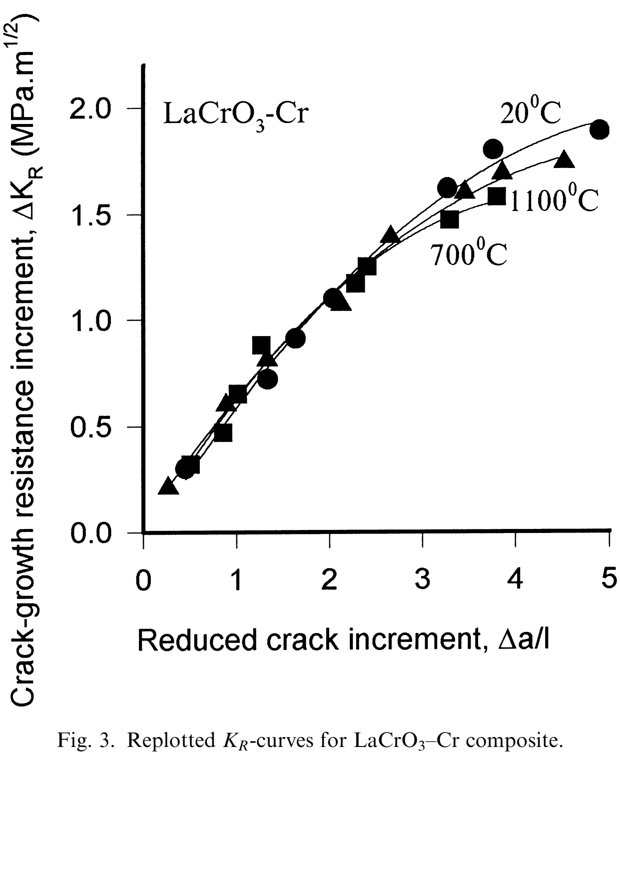


Fig. 3. Replotted $K_R$-curves for $LaCrO_3$–Cr composite.

To verify the validity of description of the $K_R$-curve behavior by the function of Eq. (5), let us consider some experimental data available on other brittle materials, the stable crack growth in which is related to the crack-face bridging in a wake zone near the crack tip. In Fig. 4 a representation of the experimental data on $K_R$-curve behavior in alumina and partially stabilized zirconia after Saadaoui, Olagnon and Fantozzi[12] is given. The data are described well by an exponential function Eq. (5).

The results of Lutz et al.,[13] for alumina ceramics seems to be a good illustration, too. Shown in Fig. 5 are the replotted $K_R$-curves using the data originated from Ref. 13 for the alumina specimens having various initial notch depths. The $K_R$-curves are fitted well when plotted versus the normalized crack increment, $\Delta a/l$. Therefore, the $\Delta K_R - \Delta a/l$ curves are notch-size independent, and can be related uniquely with the microstructure of the material. Estimations resulting from these data allow us to conclude that the value of $l$ rises with an increase in grain size (Table 1), and the corresponding increase of the length of the crack-face bridging zone can be supposed.

It follows from the above results, that the exponential function of Eq. (5) can be used as a satisfactory approximation for the $K_R$-curve behavior law in certain ceramics and ceramic-matrix composites, the stable crack growth in which is due to the crack-face bridging.

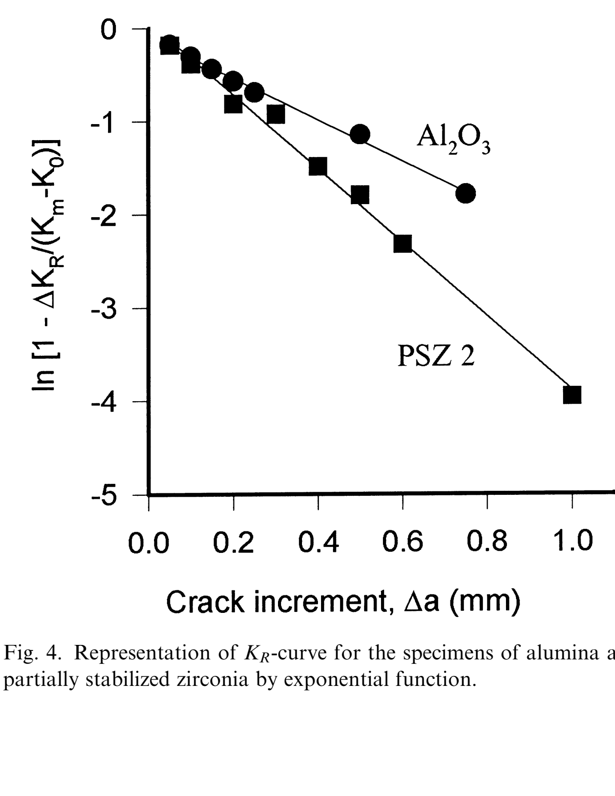


Fig. 4. Representation of $K_R$-curve for the specimens of alumina and partially stabilized zirconia by exponential function.

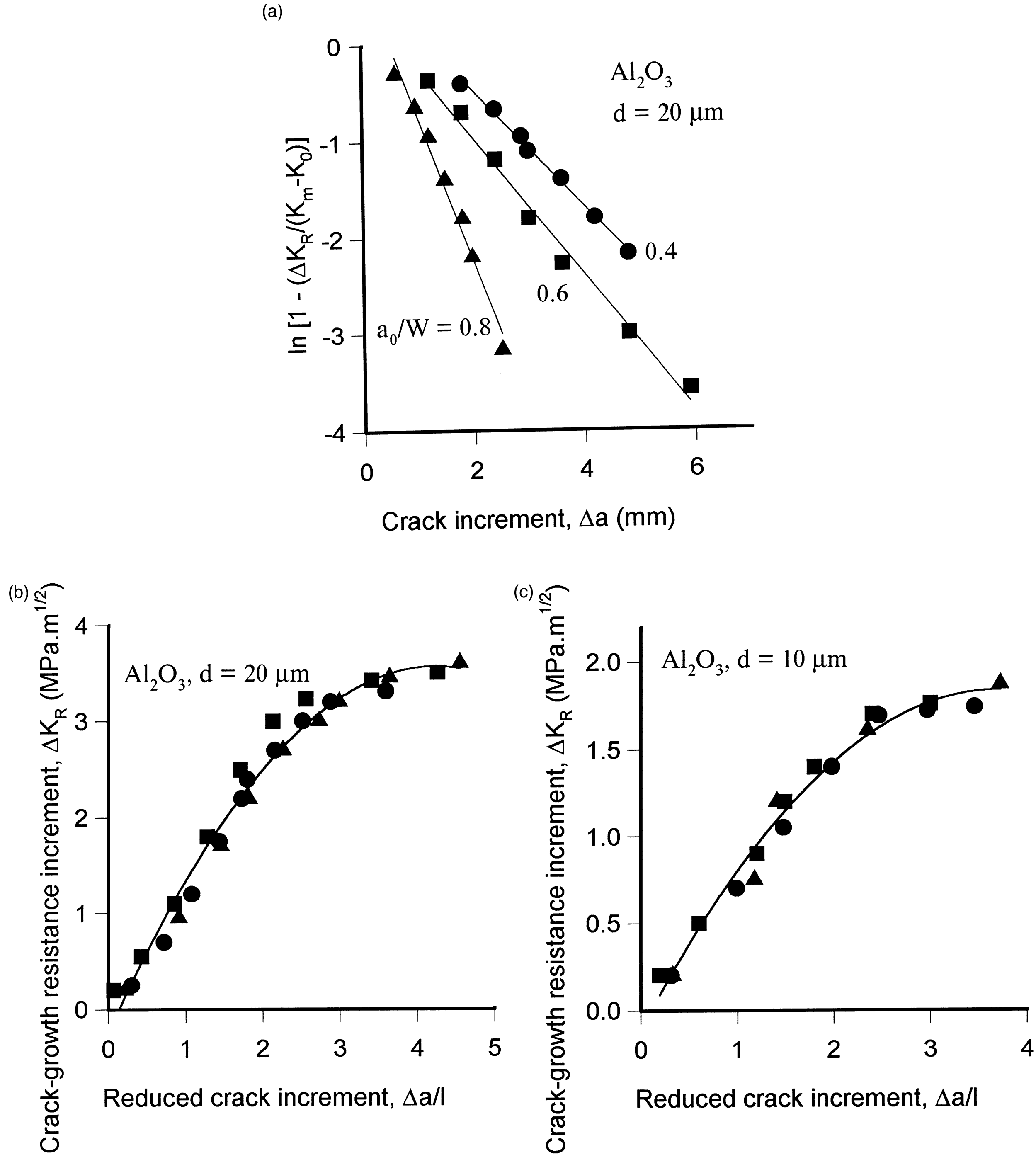


Fig. 5. Representation of $K_R$-curves for the specimens of alumina ceramics with different initial notch depth $a_0$ and mean grain size of 20 μm by exponential function (a) and replotted $K_R$-curves (b); normalized $K_R$-curves for alumina having mean grain size 10 μm (c).

The use of this function allows to eliminate the notch-size effect on the $K_R$-curve and to relate the parameters of $K_R$-curve with the microstructural features of the materials.

Various theoretical approaches to describe $K_R$-curve behaviour have been proposed.[8,13–19] Some of them are based on a concept of wake-controlled contact crack shielding.[17–19] The exponential form of the crack-growth resistance versus crack increment behavior follows from different theoretical points of view.[8,14,15] A possible substantiation of the $K_R$-curve exponential law can also be made as follows.

Let us assume that there are some ligaments in the microstructure of the material which are responsible for the crack-face bridging stresses in the wake zone. For example, these ligaments are supposed to be realated to the roughness in the interfaces resulting in friction forces during the grain-boundary sliding and the crack opening process. Let us suppose further that these ligamants are

Table 1
Estimations the normalizing factor for various aluminas

| Main grain size, $d$ (μm) | Reduced notch depth, $a_0/W$ | Normalizing factor, $l$ (μm) |
|---|---|---|
| 5 | 0.4 | 850 |
| | 0.6 | 540 |
| | 0.8 | 270 |
| 10 | 0.4 | 1210 |
| | 0.6 | 1000 |
| | 0.8 | 510 |
| 20 | 0.4 | 1670 |
| | 0.6 | 1410 |
| | 0.8 | 660 |

distributed continuously and $f(\sigma, \Delta a)d\sigma$ is the number of the ligaments that have a friction resistance in the range from $\sigma$ to $\sigma + d\sigma$ at the crack increment $\Delta a$. The number of the ligaments that were acted from the moment of the beginning of the stable crack growth to the crack increment $\Delta a$ is equal to $[f(\sigma, 0) - f(\sigma, \Delta a)]d\sigma$. An increment in the stress intensity factor due to these ligaments can be expressed as

$$\Delta K_R = \int_0^{\sigma} k(\sigma)[f(\sigma, 0) - f(\sigma, \Delta a)]d\sigma \tag{6}$$

where $k(\sigma)$ is accounted for the increment in the stress intensity factor due to a single ligament. Assuming that the change of the number of ligaments during the crack propagation is proportional to the number of the existing ligaments at this moment, i.e.

$$d[f(\sigma, 0) - f(\sigma, \Delta a)]/d\sigma = f(\sigma, \Delta a)/l \tag{7}$$

where $l$ is a normalizing factor, an expression for an increment of the stress intensity factor can be obtained by substitution of Eq. (7) into Eq. (6) and integration, if $k(\sigma)$ and $f(\sigma, \Delta a)$ are known. It is reasonable to assume that $k(\sigma) = \text{const}$, i.e. the contributions to the stress intensity factor from the ligaments are the same. It was shown that the major contribution to the bridging stress in the composite under investigation gives the debonded and sliding ligaments.[11] For such ligaments, it can be accepted that

$$f(\sigma, 0) = f_0\delta(\sigma - \sigma_0) \tag{8}$$

where $f_0 = \text{const}$ and $\delta(\sigma - \sigma_0)$ is delta-function. After substitution and integration, Eq. (5) can be obtained, where $K_m - K_0 = k(\sigma_0)f_0$.

Thus, an exponential law of the $\Delta K_R$-curve behavior follows from simple assumptions about the crack-bridging ligaments. An advantage of this function as compared to the power equation is in the clear meaning of the parameters as well as in the possibility to represent the $K_R$-curves in an invariant form.

## 4. Conclusion

The following conclusions can be drawn from the results obtained:

1. The $K_R$-curve behavior in a particulate ceramic–metal composite in the system lanthanium chromite–chromium can be described by an exponential function of Eq. (5). This is demonstrated also for other ceramic materials, such as alumina and partially stabilized zirconia.
2. The normalizing parameter $l$ of the exponential function Eq. (5) depends on testing temperature; if the value of $K_m - K_0$ does not depend on testing conditions, so the $K_R$-curves can principally be represented in an invariant form, with $\Delta K_R$ as a function of $\Delta a/l$. This allows to eliminate the notch-size effect in the $K_R$-curve testing of brittle materials.
3. The exponential law of the $K_R$-curve behavior can be substantiated assuming that the crack-face bridging ligaments are frictional with the same friction stress contribution to the total stress which prevents the crack-opening displacement.

## References


1. Steinbrech, R. W., *R*-curve behavior of ceramics. In *Fracture Mechanics of Ceramics*, Vol. 9, ed. R. C. Bradt, D. P. H. Hasselman, D. Munz, M. Sakai and V. Ya Shevchenko. Plenum, New York, 1992, pp. 187–208.
2. Hsueh, C. H. and Becher, P. F., Evaluation of bridging stress from *R*-curve behavior for nontransforming ceramics. *J. Am. Ceram. Soc*, 1988, **71**, 234–237.
3. Krause, R. F., Fuller, E. R. and Rhodes, J. F., Fracture resistance behavior of silicon carbide whisker reinforced alumina with different porosity. *J. Am. Ceram. Soc.*, 1990, **73**, 559–565.
4. Dusza, J. and Barinov, S. M., Rising crack-growth-resistance behavior of alumina and alumina–SiC whisker ceramics. *Scripta Met. Mater.*, 1993, **28**, 417–422.
5. Lutz, E. H. and Swain, M. V., Interrelation between flaw-resistance, $K_R$-curve behavior and thermal shock strength degradation in ceramics. In *Fracture Mechanics of Ceramics*, Vol. 9, ed. R. C. Bradt, D. P. H. Hasselman, D. Munz, M. Sakai and V. Ya Shevchenko. Plenum, New York, 1992, pp. 319–337.
6. Krasulin, Yu.L., Barinov, S. M. and Ivanov, V. S., Interparticle welding, mechanical properties and fracture behaviour of $LaCrO_3$–Cr cermets. *J. Mater. Sci.*, 1986, **21**, 3901–3906.
7. Atroshenko, E. S., Barykin, B. M., Ivanov, V. S., Krasulin, Yu.L. and Spiridonov, E. G., The effect of explosive compacting on the properties of 60%$LaCrO_3$–40%Cr cermet. *High Temp. — High Press.*, 1976, **8**, 21–26.
8. Broberg, K., On stable crack growth. *J. Mech. Phys. Solids*, 1975, **23**, 215–226.
9. Barinov, S. M., A compliance method for crack propagation resistance curve in brittle materials. *J. Mater. Sci. Lett.*, 1996, **15**, 2036–2037.
10. Srawley, J. E., Wide range stress intensity factor expressions for ASTM E399 standard fracture toughness specimens. *Int. J. Fract.*, 1976, **12**, 475–476.

11. Barinov, S. M., On crack-face bridging in a particulate ceramic–metal composite. *J. Mater. Sci. Lett.*, 1997, **16**, 1827–1829.
12. Saadaoui, M., Olagnon, C. and Fantozzi, G., *R*-curve behavior of alumina and PSZ at ambient and high temperatures. In *Fracture Mechanics of Ceramics*, Vol. 9, ed. R. C. Bradt, D. P. H. Hasselman, D. Munz, M. Sakai and V. Ya Shevchenko. Plenum, New York, 1992, pp. 357–369.
13. Lutz, E. H., Reichl, A. and Steinbrech, R. W., Size dependence of the *R*-curve of quasi-brittle materials. In *Fracture Mechanics of Ceramics*, Vol. 11, ed. R. C. Bradt, D. P. H. Hasselman, D. Munz, M. Sakai, and V. Ya Shevchenko, Plenum, New York, 1996, pp. 53–62.
14. Cherepanov, G. P., *Mechanics of Brittle Fracture*. Science, Moscow, 1974.
15. Barinov, S. M. and Shevchenko, V. Ya., Universal *R*-curve of crack propagation resistance in ceramic composites. In *Fracture Mechanics of Ceramics*, Vol. 9, ed. R. C. Bradt, D. P. H. Hasselman, D. Munz, M. Sakai and V.Ya Shevchenko, Plenum, New York, 1992, pp. 209–217.
16. Cox, B. N. and Marshall, D. B., The determination of crack bridging forces. *Int. J. Fract.*, 1991, **49**, 159–176.
17. Cox, B. N. and Marshall, D. B., Stable and unstable solutions for bridged cracks in various specimens. *Acta Met.*, 1991, **39**, 579–589.
18. Suzuki, T. and Sakai, M., A model for crack-face bridging. *Int. J. Fract.*, 1994, **65**, 329–344.
19. Fett, T., Evaluation of the bridging relation from crack-opening-displacement measurements by use of the weight function. *J. Am. Ceram. Soc.*, 1995, **78**, 945–948.